# Technical Comments

## Comment on the Effect of Sudden Compressions on the Turbulent Boundary Layer

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### Nomenclature

$C$ = Crocco number adjacent to boundary layer $C = u_e/(2C_pT_{oe})^{1/2}$
$H_i$ = incompressible boundary-layer shape parameter immediately upstream of separation
$M$ = Mach number
$u$ = velocity
$x, y$ = coordinates shown in Fig. 1b
$\gamma$ = ratio of specific heats
$\delta$ = boundary-layer thickness
$\delta^{**}$ = boundary-layer momentum thickness
$\theta_w$ = separation shock angle (see Fig. 1b)
$\theta_2$ = separated flow direction (see Fig. 1b)

***Subscripts***

$e$ = external flow adjacent to boundary layer
$i$ = incompressible
$F$ = flow conditions immediately downstream of separation
$I$ = flow conditions immediately upstream of separation

IN a recent article, White[1] presented a method for calculating the turbulent boundary-layer momentum thickness change across sudden expansions and sudden compressions. White's analysis for the sudden expansion case shows excellent agreement with other analyses of the problem. The purpose of this note is to point out some striking differences between the predictions of White,[1] the present author,[2] and Childs, Paynter and Redecker[3] concerning the effect of sudden compressions on the momentum thickness.

The momentum thickness change across a sudden compression is important to the analysis of separating and separated boundary layers. Once the momentum thickness is known downstream of separation, integral methods such as those discussed by White[1] can be used to determine the overall flowfield.

White's flow model for the sudden compression case is shown in Fig. 1a. White assumes that the compression is a gradual isentropic turning of the flow, and can be described by the boundary-layer momentum integral equation neglecting the wall friction term. White uses Culick and Hill's[4] representation for the effect of compressibility on the boundary-layer shape parameter to obtain the following expression for the momentum thickness change:

$$\frac{\delta_F^{**}}{\delta_I^{**}} = \left(\frac{C_I}{C_F}\right)^{(2+H_i)} \left(\frac{C_F^2-1}{C_I^2-1}\right)^{(1/2)\{H_i+1-[2/(\gamma-1)]\}} \tag{1}$$

Schlieren photographs of supersonic turbulent boundary-layer separation[2,5] indicate that the separation shock penetrates far into the boundary layer as a strong shock. The strong transverse pressure gradients in the boundary layer due to this shock makes it seem doubtful that the boundary-layer equations are valid in this region. Therefore, consideration has been given to a new flow model, shown in Fig. 1b, which does not require the use of the boundary-layer equations through the abrupt compression.† The separation shock is considered to penetrate the boundary layer nearly to the wall, and the pressure and flow direction are assumed uniform on either side of the shock.

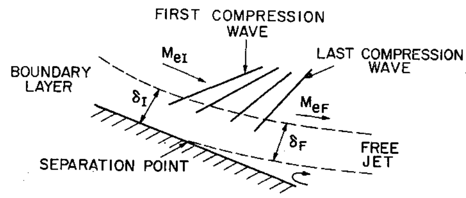


**Fig. 1a White's model for abrupt compression.**

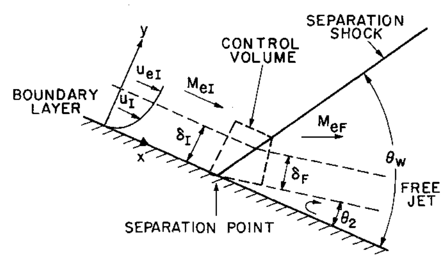


**Fig. 1b Separation shock model for abrupt compression.**

Applying continuity to the control volume of Fig. 1b, assuming an irreversible oblique shock compression, and assuming that the velocity profile is not discontinuous at the separation point (this implies that the incompressible shape factor $H_i$ is a constant across the separation point) enables calculation of the momentum thickness change. The following expression for the momentum thickness change is obtained using Culick and Hill's shape parameter[2]:

$$\frac{\delta_F^{**}}{\delta_I^{**}} = \frac{\left[H_i + \left(\frac{\gamma-1}{2}\right) M_{eI}^2(H_i+1)\right]\sin(\theta_w-\theta_2)}{\left[H_i + \left(\frac{\gamma-1}{2}\right) M_{eF}^2(H_i+1)\right]\sin\theta_w} \tag{2}$$

Childs, Paynter and Redecker[3] also use the flow model of Fig. 1b. They apply continuity and the momentum equation in the $x$ direction to the control volume. By rearranging the momentum thickness change obtained in Ref. 3 and introducing Culick and Hill's shape parameter, the following expression for the momentum thickness change is obtained:

$$\frac{\delta_F^{**}}{\delta_I^{**}} = \frac{\sin(\theta_w-\theta_2)}{\sin\theta_w}\left\{\left[\frac{\cos(\theta_w-\theta_2)}{\cos\theta_2\cos\theta_w}\right] + \left[H_i + \left(\frac{\gamma-1}{2}\right) M_{eI}^2(H_i+1)\right]\left[\frac{\cos(\theta_w-\theta_2)}{\cos\theta_2\cos\theta_w} - 1\right]\right\} \tag{3}$$

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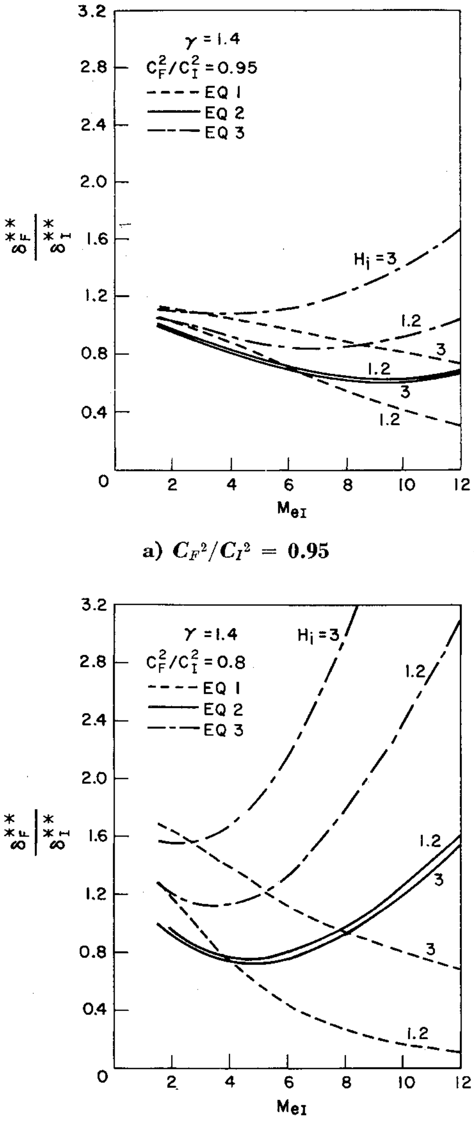
† However, the boundary-layer equations are used to calculate the boundary-layer growth upstream of the separation point.




a) $C_F^2/C_I^2 = 0.95$

b) $C_F^2/C_I^2 = 0.80$

**Fig. 2 Effect of an abrupt compression on boundary-layer momentum thickness.**

Equations (1–3) are plotted in Figs. 2a and 2b. These plots were made for two compression ratios ($C_F^2/C_I^2 = 0.95$ and 0.80) and two values of the incompressible shape factor ($H_i = 1.2$ and $H_i = 3$).‡ $\theta_w$ and $\theta_2$ are calculated for a given compression ratio and initial Mach number by using the standard oblique shock relationships.

As shown in Fig. 2, Eq. (1) predicts a monotonic decrease in $\delta_F^{**}/\delta_I^{**}$ as $M_{eI}$ increases for a fixed compression ratio $C_F^2/C_I^2$. Equations (2) and (3) both show a different qualitative trend, i.e., $\delta_F^{**}/\delta_I^{**}$ decreases at first and then increases as $M_{eI}$ increases for a fixed compression ratio.

All three equations show qualitative agreement with White's experiments[1] at a compression ratio of approximately 0.87, where $\delta_F^{**}/\delta_I^{**}$ was between 1.11 and 0.99 for $M_{eI}$ between 2.5 and 2.9. The fact that Eq. (2) is relatively insensitive to the choice of $H_i$ (an unknown) makes it more attractive than Eqs. (1) and (3) which are quite sensitive to the chosen $H_i$. However, from Fig. 2 it is obvious that further experiments are needed to select the superior approximation.

‡ The actual value of $H_i$ at separation is unknown for supersonic flow. However, these limits should cover the range of possible values since Schlicting[6] gives the following for incompressible flow; $1.8 \leq H_i \leq 2.4$.

## References


[1] White, R. A., "Effect of sudden expansions or compressions on the turbulent boundary layer," AIAA J. **4**, 2232–2234 (1966).

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[4] Culick, R. E. C. and Hill, J. A. F., "A turbulent analog of the Stewartson-Illingworth transformation," J. Aeronaut. Sci. **25**, 259–262 (April 1958).

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# Time Dilatation in Relativity


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The article on time dilatation by v. Krzywoblocki[1] in the AIAA Journal, in which the author claims to resolve the clock paradox, calls for the following comments. His arguments are based mainly on the following three points: 1) The possibility of two processes, the Clausius process and the Lorentz process, the independence of the former frame from relativistic transformations. 2) The internal mechanism of a clock belongs to a Clausius family; i.e., the light signals do not affect the action or the rate of action of the mechanism of the clock. 3) Space trip clocks may be constructed with a different scale, i.e., a unit of time $\gamma^{-1}$ times that of the earth clock.

Assumption 1 is ruled out in relativity theory, as no absolute frames or absolute processes independent of the frames of references, are supposed to exist. If assumption 2 is valid, that is, if the mechanism is not affected, it is not correct to introduce 3. Therefore, 2 and 3 appear to be contradictory.

We may now proceed with the following comments:

a) That the rate of clock should be adjusted to coincide with a fixed clock is valid in pre-Einstein theory only, since absolute time is itself ruled out.

b) No reference appears anywhere in the discussion of time dilatation about the effects of velocity or acceleration on the movement or mechanism of a clock.

c) For experiments involving high velocities, only atomic clocks, such as the vibration of the Caesium atom, are used. The possibility of adjustment of rate does not arise.

d) If the clock in $S'$ is retarded relative to $S$, then the clock in $S$ is retarded relative to $S'$; the adjustment of one does not solve the riddle.

e) If $S$ moves with velocity $v$ relative to $S_1$, and with velocity $v_2$ relative to $S_2$, and if $S$, with the adjusted clock, leaves $S_1$ and after some time passes through $S_2$, then the synchronization of $S_1$ and $S_2$ will be complicated as it is not working at a normal rate.

f) Finally, the question of adjustment assumes prepared journeys, as in the case of space travel. But in physics we do not deal with prepared journeys, but will have to explain the relations between the journeys, not necessarily planned by us.

It is better that the paradox is left as it is, as all the experimental evidence has very well supported Einstein's special theory of relativity. The solution of the paradox does not lie


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